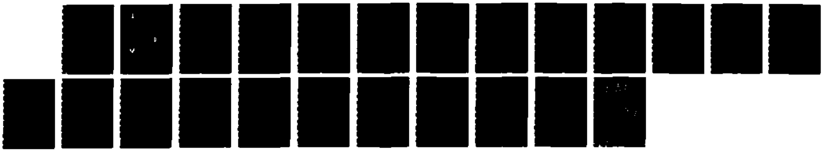
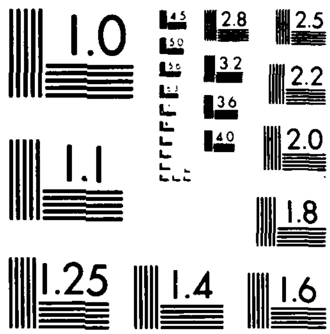
AD-A184 200 TORSION AS A SOURCE OF EXPANSION IN A BIANCHI TYPE I UNIVERSE IN THE SELF (U) ARMY MISSILE COMMAND REDSTONE ARSENAL AL GUIDANCE AND CONTROL J C BRADAS JUN 87 1/1

UNCLASSIFIED AMSMI/TR-RD-GC-87-3 SBI-AD-E951 057 F/G 3/2 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

TECHNICAL REPORT RD-GC-87-3

TORSION AS A SOURCE OF EXPANSION IN A BIANCHI TYPE I UNIVERSE IN THE SELF-CONSISTENT EINSTEIN-CARTAN THEORY OF A PERFECT FLUID WITH SPIN DENSITY

James C. Bradas, et al.
Guidance and Control Directorate
Research, Development, & Engineering Center

JUNE 1987

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898-5000

Approved for public release; distribution is unlimited.

DISPOSITION INSTRUCTIONS

DESTROY THIS REPORT WHEN IT IS NO LONGER NEEDED. DO NOT RETURN IT TO THE ORIGINATOR.

DISCLAIMER

THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.

TRADE NAMES

USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL INDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188
Exp. Date Jun 30, 1986

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution is unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S): RD-GC-87-3	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION: Guidance & Control Directorate — 6b. OFFICE SYMBOL (If applicable): AMSMI-RD-GC-T	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code): Commander, US Army Missile Command, ATTN: AMSMI-RD-GC-T, Redstone Arsenal, AL 35898-5254	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION — 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO. / PROJECT NO / TASK NO / WORK UNIT ACCESSION NO

11. TITLE (Include Security Classification)
TORSION AS A SOURCE OF EXPANSION IN A BIANCHI TYPE I UNIVERSE IN THE SELF-CONSISTENT EINSTEIN-CARTAN THEORY OF A PERFECT FLUID WITH SPIN DENSITY

12. PERSONAL AUTHOR(S)

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Final	FROM ______ TO Sep 86	JUNE 1987	22

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	Gravitation	Torsion
			General Relativity	Spin
			Cosmology	Einstein-Cartan Theories

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

We show that a generalized (or "power law") inflationary phase arises naturally and inevitably in a simple (Bianchi Type I) anisotropic cosmological model in the self-consistent Einstein-Cartan gravitation theory with the improved stress-energy-momentum tensor with spin density of Ray and Smalley. This is made explicit by analytical solution of the field equations of motion of the fluid variables. The inflation is caused by the angular kinetic energy density due to spin. The model further elucidates the relationship between fluid vorticity, the angular velocity of the inertially-dragged tetrads, and the precession of the principal axes of the shear ellipsoid. Shear is not effective in damping the inflation.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: James C. Bradas	22b. TELEPHONE (Include Area Code): (205) 876-8459 — 22c. OFFICE SYMBOL: AMSMI-RD-GC-T

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to Alphonsus J. Fennelly of Teledyne Brown Engineering and Larry L. Smalley of the Department of Physics, University of Alabama in Huntsville, for their cooperation on this project.

TABLE OF CONTENTS

I. INTRODUCTION

The inflationary universe model, in which expansion is accelerated ($\ddot{R} > 0$), is expected to occur in the early universe containing matter in the form of bare quantum fields, the expansion being exponential (R exp[kt]) [1]. Power-law inflation, in which the expansion scale factor obeys a power-law relation with the time, is also a possible result of those physical processes in the early universe. It is important because it too will solve such cosmological problems as horizons, homogeneity, and flatness [2].

Further, inflation is important because it is thought that it can solve the problem of the apparent large-scale isotropy of the universe [3]. This is so because its presence minimizes the acceleration produced by the existence of a cosmological constant, which isotropizes the universe [4]. There is, however, the model in the Einstein-Cartan gravitational theory with the Ray-Smalley improved stress-energy-momentum tensor (SEMT) with spin. Gasperini has proven that for the case of an rms spin density [5]. However, it will be shown that the inflationary epoch occurs because of the density of spin angular kinetic energy, which is a local quantity dependent on the spin, and therefore, it is not necessary to resort to an rms expectation value of the spin density operator to generate the spin terms necessary to induce inflation. The model used is a sample anisotropic (Bianchi Type I) cosmological model with shear, but with vanishing spatial curvature (Euclidean model). Formal solutions of the Einstein-Cartan equations and of the fluid equations of motion are exhibited and shown to lead to conditions producing an inflationary epoch in the very early universe. The inflation is due to the angular kinetic energy density of the spin. The shear is not effective in preventing or damping the inflation in the model. The model further brings out the relationship between the fluid vorticity, the angular velocity of observers' inertially-dragged and Fermi-transported reference tetrads, the precession of the principal axes of the ellipsoid of the shear rate, and the torsion. It concludes with comments of further work and suggestions for new investigations of this and related models.

In Section II, the basic equations of the model, following the pattern of Bianchi I spacetime, are given, and its behavior is shown in a Riemann-Cartan spacetime in Section III. The conclusions are presented in Section IV.

II. FORM OF THE MODEL

In this paper, the metric chosen has the following form, as used by Misner [6];

$$ds^2 = -dt^2 + e^{2\alpha} e^{2\beta}_{ij}\, dx^i\, dx^j \tag{1}$$

where α is a scalar function of time and $\exp[\beta]$ a traceless, 3 x 3 matrix, also a function of time. Following the method of differential forms, Cartan's first equations is written, connecting the basis forms with Torsion $S_{\mu\nu}{}^{a}$, as

$$d\omega^{\alpha} + \omega^{\alpha}{}_{\nu} \wedge \omega^{\nu} = \tfrac{1}{2} S_{\mu\nu}{}^{\alpha}\, \omega^{\mu} \wedge \omega^{\nu} \tag{2}$$

where the torsion is defined as

$$S_{\mu\nu}{}^{\alpha} = \Gamma^{\alpha}_{[\mu\nu]} - \tfrac{1}{2} C_{\mu\nu}{}^{\alpha}. \tag{3}$$

Here, the torsion is defined as the true antisymmetric portion of the affine connection (non-zero in a holonomic frame), with the C's the antisymmetric portion (if any) due to the choice of tetrads. Choosing a basis one-form set which diagonalizes the above metric and puts it into Minkowski form, gives:

$$\omega^{0} = dt \tag{4a}$$

$$\omega^{i} = e^{\alpha} e^{\beta}_{ij}\, dx^{j} \tag{4b}$$

In a Bianchi Type I cosmology, all the spatial structure constants are zero. Thus,

$$C^{i}{}_{jk} = 0 \tag{5}$$

The tetrads have the following properties: The capital Latin indices are used to refer to anholonomic coordinates, and Greek indices to holonomic coordinates. Thus,

$$E_{A\mu}\, E_{B}{}^{\mu} = g_{AB} \tag{6a}$$

$$E^{A}{}_{\mu}\, E_{A\nu} = \eta_{\mu\nu} \tag{6b}$$

Following Ray and Smalley [7], an expression giving the orientation of the spin-density $s_{\mu\nu}$, angular momentum $w_{\mu\nu}$ of the tetrads, and the improved SEMT is written as

$$s_{\mu\nu} = k(x)\, (E^{1}{}_{\mu}\, E^{2}{}_{\nu} - E^{1}{}_{\nu}\, E^{2}{}_{\mu}). \tag{7a}$$

$$w_{\mu\nu} = \tfrac{1}{2}[D(E^{A}{}_{\mu})\, E_{A\nu} - D(E^{A}{}_{\nu})\, E_{A\mu}] \tag{7b}$$

$$\begin{aligned} T^{\alpha\beta}_{\text{Ray-Smalley}} &= \rho(1 + \varepsilon + P/\rho)\, u^{\alpha} u^{\beta} + P\, g^{\alpha\beta} \\ &\quad + 2\rho\, D(u_{\nu})\, u^{(\alpha} s^{\beta)\nu} + \overset{*}{\nabla}_{\nu}[\rho u^{(\alpha} s^{\beta)\nu}] \\ &\quad - \rho w_{\nu}{}^{(\alpha} s^{\beta)\nu} \end{aligned} \tag{7c}$$

Using tetrads consistent with the choice of Bianchi Type I (Ref. 8, page 110) structure in Eq. (7a), the non-zero components of $s_{\mu\nu}$ are

$$s_{12} = k(x) = -S_{21} \tag{8}$$

Again, following Ray and Smalley [7], the trace-free (proper) torsion $S_{\alpha\beta}{}^{\mu}$, which is defined as

$$\hat{S}_{\mu\nu}{}^{\alpha} = S_{\mu\nu}{}^{\alpha} + (2/3)\delta^{\alpha}_{[\mu}S_{\nu]\beta}{}^{\beta} \tag{9}$$

is related to the spin density $s_{\mu\nu}$ by the relationship

$$\hat{S}_{\mu\nu}{}^{\alpha} = \tfrac{1}{2}\kappa\rho s_{\mu\nu}\, u^{\alpha} \quad . \tag{10}$$

where $K = 8\pi G$, and G is the gravitational constant. For purposes of this paper, a co-moving frame ($u^{\alpha} = \delta^{\alpha}_{o}$) with normalized four-velocity ($u_{\alpha}u^{\alpha} = -1$) is used. Thus, the non-zero components of trace-free torsion are:

$$\hat{S}_{12}{}^{o} = \tfrac{1}{2}\kappa\rho s_{12}u^{o} = -S_{21}{}^{o}. \tag{11}$$

which gives a relationship between the torsion and proper torsion:

$$S_{12}{}^{o} = \hat{S}_{12}{}^{o} = \tfrac{1}{2}\kappa\rho k(x)u^{o} = -S_{21}{}^{o} \tag{12}$$

These values of torsion will be used in the model under consideration.

Proceeding with the calculation, the connection two-forms are calculated using Cartan's first equation. In this paper, the following notation for derivatives are used: The overdot indicates partial differentiation with respect to time, as in

$$\dot{A}_{ij\ldots} = \partial/\partial t\, A_{ij\ldots} \text{ or } (A_{ij\ldots}) \, . \tag{13a}$$

while the directional covariant derivative along the four veloctiy is indicated by

$$D(A_{ij\ldots}) = A_{ij\ldots;a}\, u^{a} \tag{13b}$$

The quantity

$$e^{\beta}{}_{ij} \cdot e^{-\beta j}{}_{k}, \tag{14}$$

which occurs when doing the computations involved in Cartan's First Equation, is split into a symmetric and antisymmetric part, the former related to the shear, the latter related to the twist of the congruence of the normals to the homogeneous hypersurfaces. These are written

$$\bar{\sigma}_{ik} = e^{\beta}{}_{(i|j|} \cdot e^{-\beta j}{}_{k)} \tag{15a}$$

$$\bar{\tau}_{ik} = e^{\beta}{}_{[i|j|} \cdot e^{-\beta j}{}_{k]} \tag{15b}$$

The connection two forms are summized in Table 1 below.

TABLE 1. Connection Two-Forms

$\bar{\omega}^{o}{}_{j}$	$(\dot{\alpha}\,\delta_{jk} + \bar{\sigma}_{jk} + \frac{1}{2}S_{jk}{}^{o})\,\omega^{k}$
$\bar{\omega}^{j}{}_{o}$	$(\dot{\alpha}\,\delta_{jk} + \bar{\sigma}_{jk})\,\omega^{k}$
$\bar{\omega}^{j}{}_{k}$	$-\bar{\tau}_{jk}\,dt$
$\bar{\omega}^{o}{}_{o}$	0
$\bar{\omega}^{i}{}_{i}$	0

Using the connection two-forms summarized above, Cartan's Second Equation is computed as

$$\bar{\Theta}_{\beta}{}^{\alpha} = d\bar{\omega}_{\beta}{}^{\alpha} + \bar{\omega}_{\nu}{}^{\alpha} \wedge \bar{\omega}_{\beta}{}^{\nu}$$
$$= \tfrac{1}{2}R^{\alpha}{}_{\beta\mu\nu}\,\omega^{\mu} \wedge \omega^{\nu} \quad (16)$$

Doing the computations involved in Eq. (16) are somewhat tedious, and only the results are stated. The non-zero components of the Riemann tensor are ($\tilde{k}_{\ell j} = \tilde{\sigma}_{\ell j} + \tilde{\tau}_{\ell j}$);

$$\bar{R}^{i}{}_{ooj} = -\bar{R}^{i}{}_{ojo}$$
$$= \ddot{\alpha}\,\delta_{ij} + \dot{\bar{\sigma}}_{ij} + (\dot{\alpha} + \bar{\sigma})^{2}_{ij} + [\bar{\sigma} \cdot \bar{\tau}]_{ij} \quad (17a)$$

$$\bar{R}^{i}{}_{ooo} = \bar{R}^{i}{}_{ojk} = \bar{R}^{o}{}_{ooo} = 0 \quad (17b)$$

$$\bar{R}^{o}{}_{ioj} = -\bar{R}^{o}{}_{ijo}$$
$$= \ddot{\alpha}\,\delta_{ij} + \dot{\bar{\sigma}}_{ij} + (\dot{\alpha} + \bar{\sigma})^{2}_{ij} + [\bar{\sigma} \cdot \bar{\tau}]_{ij}$$
$$+ \tfrac{1}{2}(\dot{S}_{ij}{}^{o} + \dot{\alpha}S_{ij}{}^{o} + \tilde{k}_{\ell j}S_{i}{}^{\ell o} - \bar{\tau}_{\ell i}\,S_{j}{}^{\ell o}) \quad (17c)$$

$$\bar{R}^i{}_{jk\ell} = (\dot{\alpha}\, \delta^i{}_k + \bar{\sigma}^i{}_k)\, (\dot{\alpha}\, \delta_{j\ell} + \bar{\sigma}_{j\ell}) - (\dot{\alpha}\, \delta^i{}_\ell + \bar{\sigma}^i{}_\ell)\, (\dot{\alpha}\, \delta_{jk} + \bar{\sigma}_{jk})$$

$$+ \tfrac{1}{2}[S_{j\ell}{}^o\, (\dot{\alpha}\, \delta^i{}_k + \bar{\sigma}^i{}_k) - S_{jk}{}^o\, (\dot{\alpha}\, \delta^i{}_\ell + \bar{\sigma}^i{}_\ell)] \tag{17d}$$

Using the Reimann tensor components, shown above, the Ricci tensor components $R_{\mu\nu}$ are then calculated by contracting the first and third indices of the Riemann tensor. The non-zero components of the Ricci tensor are

$$\bar{R}_{oi} = \bar{R}_{io} = 0,$$

$$\bar{R}^o{}_o = 3\, \ddot{\alpha} + 3\, (\dot{\alpha})^2 + \bar{\sigma}_{ij}\, \bar{\sigma}^{ij}\ , \tag{18a}$$

$$\bar{R}_{ij} = (\ddot{\alpha}\, \delta^o_{ij} + \dot{\bar{\sigma}}_{ij}) + 3\, \dot{\alpha}\, (\dot{\alpha}\, \delta_{ij} + \bar{\sigma}_{ij}) + [\bar{\sigma} \cdot \bar{\sigma}]_{ij}$$

$$+ \tfrac{1}{2}[\dot{S}_{ij}{}^o + 3\dot{\alpha}\, S_{ij}{}^o - 2\bar{\sigma}_{k[i}S_{j]}{}^{ko}]\ . \tag{18b}$$

The curvature scalar R is

$$\bar{R} = 6\, \ddot{\alpha} + 12\, \dot{\alpha}^2 + \bar{\sigma}_{ij}\, \bar{\sigma}^{ij} \tag{19}$$

The 16 components of the Einstein tensor are defined by

$$\bar{G}_{\mu\nu} = \bar{R}_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\, \bar{R}\ , \tag{20}$$

and summarized in Table 2. For brevity sake, the additional terms have been written explicitly only due to torsion. The Einstein tensor which is obtained in the regular GR theory is listed only symbolically (see Misner [6] for regular GR components).

TABLE 2. Summarization of the Einstein Tensor Components

μ	ν	$G_{\mu\nu}$ (Unprimed G's are standard results from GR)
0	0	G_{00}
0	i	0 (i = 1,2,3)
i	0	0 (i = 1,2,3)
1	1	G_{11}
1	2	$G_{12} + \frac{1}{2}[\dot{S}_{12}{}^{o} + 3\,\dot{\alpha}\,S_{12}{}^{o}]$
1	3	$G_{13} + \frac{1}{2}\bar{\tau}_{23}\,S_{12}{}^{o}$
2	1	$G_{21} + \frac{1}{2}[\dot{S}_{21}{}^{o} + 3\,\dot{\alpha}\,S_{21}{}^{o}]$
2	2	G_{22}
2	3	$G_{23} + \frac{1}{2}\bar{\tau}_{13}\,S_{21}{}^{o}$
3	1	$G_{31} - \frac{1}{2}\bar{\tau}_{23}\,S_{12}{}^{o}$
3	2	$G_{32} - \frac{1}{2}\bar{\tau}_{13}\,S_{21}{}^{o}$
3	3	G_{33}

To calculate the Ray-Smalley tensor components in this model, the affine connections will be needed, which are contained implicitly in Table 1. Using the relationship that $\omega^{\alpha}{}_{\gamma} = \Gamma^{\alpha}{}_{\beta\gamma}\,\omega^{\beta}$ determines:

$$\Gamma^{o}_{ij} = \dot{\alpha}\,\delta_{ij} + \bar{\sigma}_{ij} - \tfrac{1}{2}S_{ij}{}^{o} \tag{21a}$$

$$\Gamma^{i}_{jo} = \dot{\alpha}\,\delta^{i}{}_{j} + \bar{\sigma}^{i}{}_{j} \tag{21b}$$

$$\Gamma^{i}_{oj} = -\bar{\tau}^{i}{}_{j} \tag{21c}$$

$$\Gamma^{i}_{jk} = 0 \tag{21d}$$

$$\Gamma^{o}_{oo} = \Gamma^{o}_{oi} = 0 \tag{21e}$$

$$\Gamma^{i}_{io} = \Gamma^{i}_{oo} = 0 \tag{21f}$$

The coupling of the Einstein tensor in the Einstein-Cartan theory with the Ray-Smalley Improved SEMT involves writing the field equations in the so-called self-consistent form;

$$\bar{G}^{\alpha\beta} - \overset{*}{\nabla}_{\mu}(Q^{\alpha\beta\mu} - Q^{\beta\mu\alpha} + Q^{\mu\alpha\beta}) = \kappa T^{\alpha\beta}_{\text{Ray-Smalley}} \tag{22}$$

where

$$\overset{*}{\nabla}_{\nu}(\quad) = \nabla_{\nu}(\quad) + 2S_{\nu\alpha}{}^{\alpha}(\quad) \tag{23a}$$

and $Q_{\alpha\beta}{}^{\mu}$ is the modified torsion, defined as

$$Q_{\alpha\beta}{}^{\mu} = S_{\alpha\beta}{}^{\mu} + 2\,\delta^{\mu}_{[\alpha}S_{\beta]\nu}{}^{\nu} \tag{23b}$$

Because the mass-conserving case is being considered, the form of the torsion in this model becomes

$$Q_{\alpha\beta}{}^{\mu} = S_{\alpha\beta}{}^{\mu} = \hat{S}_{\alpha\beta}{}^{\mu} \tag{23c}$$

and $\overset{*}{\nabla}_{\nu}(\ ) = \nabla_{\nu}(\ )$.

The field equations generated in the model versus the values of α and β, are summarized in Table 3.

TABLE 3. Field Equations

$$\bar{G}^{\alpha\beta} - \nabla^{*}_{\mu} (Q^{\alpha\beta\mu} - Q^{\beta\mu\alpha} + Q^{\mu\alpha\beta}) = \kappa T^{\alpha\beta}_{\text{Ray-Smalley}}$$

α	β	Field Equations
0	0	$3\,\dot{\alpha}^2 - (\frac{1}{2}\bar{\sigma}_{ij}\,\bar{\sigma}_{ij} = \kappa\rho(1 + \varepsilon)$
0	i	$0 = 0 \quad i = 1,2,3$
i	0	$0 = 0 \quad i = 1,2,3$
1	1	$-2\ddot{\alpha} - 3(\dot{\alpha})^2 - \frac{1}{2}\bar{\sigma}_{ij}\,\bar{\sigma}_{ij} + \overset{\blacktriangle}{\sigma}_{11} + [\bar{\sigma}, \bar{\tau}]_{11} = \kappa P + \kappa\rho\bar{\tau}_{21}s^{12}$
1	2	$\overset{\blacktriangle}{\sigma}_{12} + 3\,\bar{\sigma}_{12}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{12} - \frac{1}{2}[\dot{S}_{12}{}^{o} + 3\,\dot{\alpha}\,S_{12}{}^{o}] = 0$
1	3	$\dot{\alpha}_{12} + 3\,\bar{\sigma}_{13}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}_{13} - \frac{1}{2}S_{12}{}^{o}\,\bar{\tau}_{23} = \frac{1}{2}\dot{\kappa}\rho\bar{\tau}_{23}s^{12}$
2	1	$\overset{\blacktriangle}{\sigma}_{21} + 3\,\bar{\sigma}_{21}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{21} - \frac{1}{2}[\dot{S}_{21}{}^{o} + 3\,\dot{\alpha}\,S_{21}{}^{o}] = 0$
2	2	$-2\,\ddot{\alpha} - 3(\dot{\alpha})^2 - \frac{1}{2}\bar{\sigma}_{ij}\,\bar{\sigma}ij + \overset{\blacktriangle}{\sigma}_{22} + [\bar{\sigma}, \bar{\tau}]_{22} = \kappa P + \kappa\rho\bar{\tau}_{12}s^{21}$
2	3	$\overset{\blacktriangle}{\sigma}_{23} + 3\,\bar{\sigma}_{23}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{23} - \frac{1}{2}S_{21}{}^{o}\,\bar{\tau}_{13} = \frac{1}{2}\kappa\rho\bar{\tau}_{13}s^{21}$
3	1	$\overset{\blacktriangle}{\sigma}_{31} + 3\,\bar{\sigma}_{31}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{31} - \frac{1}{2}S_{12}{}^{o}\,\bar{\tau}_{23} = \frac{1}{2}\kappa\rho\bar{\tau}_{23}s^{12}$
3	2	$\overset{\blacktriangle}{\sigma}_{32} + 3\,\bar{\sigma}_{32}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{32} - \frac{1}{2}S_{21}{}^{o}\,\bar{\tau}_{13} = \frac{1}{2}\kappa\rho\bar{\tau}_{13}s^{21}$
3	3	$-2\,\ddot{\alpha} - 3(\dot{\alpha})^2 - \frac{1}{2}\bar{\sigma}_{ij}\,\bar{\sigma}^{ij} + \overset{\blacktriangle}{\sigma}_{33} + [\bar{\sigma}, \bar{\tau}]_{33} = \kappa P$

The equation [00], at first appearance, seems to contain no spin energy terms. However, the energy term is there by virtue of the term, ε, located on the right hand side of the equation. Recalling the thermodynamic laws of the fluid, as presented by Ray and Smalley [7], the differential of energy $d\varepsilon$ is given by:

$$d\varepsilon = T\,ds - P\,d(1/\rho) + \frac{1}{2}w_{\alpha\beta}\,ds^{\alpha\beta}\,. \tag{24}$$

Although the specific form of each of the thermodynamic variables for the fluid is unknown in this model, it may be stated that the integrated energy ε represents a correction to the standard energy term usually written in the standard theory of perfect fluids. The usual term for the T_{oo} energy density components of the SEMT is simply $\kappa\rho$. In this model, it is assumed that the correction term ε is small compared to the total energy due to fluid density. Thus, in the scaling laws developed in the next section, $\rho' = \rho\,(1 + \varepsilon)$ will have the same scaling behavior as ρ.

III. BEHAVIOR OF THE MODEL WITH TORSION

From the field equations contained in Table 3, the following observations are made. The components [00], [0i], and [i0] are identical to their GR counterparts. Torsion appears in all the other field equations. In order to determine the effect of torsion on the solutions to the field equations, self-consistency must be demanded. To elucidate, consider equations [12] and [21]. These two are related since the GR terms involve shear terms which are symmetric, the expansion which is a scalar, and the antisymmetric product of shear and antisymmetric product of shear and vorticity. By examining and writing out equations [12] and [21] in more detail, they become

$$[12]:\quad \dot{\bar{\sigma}}_{12} - 3\,\bar{\sigma}_{12}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{12} - \tfrac{1}{2}[\dot{S}_{12}{}^{o} + 3\dot{\alpha}S_{12}{}^{o}] = 0 \tag{25a}$$

$$[21]:\quad \dot{\bar{\sigma}}_{21} + 3\,\bar{\sigma}_{21}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{21} - \tfrac{1}{2}[\dot{S}_{21}{}^{o} + 3\,\dot{\alpha}\,S_{21}{}^{o}] = 0 \tag{25b}$$

Reversing the indices in Eq. (25b), an equation identical to Eq. (25a) is obtained, except for the terms involving torsion. (It can be shown that $[\tilde{\sigma}, \tilde{\tau}]_{ij} = [\tilde{\sigma}, \tilde{\tau}]_{ji}$.) Doing this operation, Eq. (25b) becomes

$$[21]:\quad \dot{\bar{\sigma}}_{12} + 3\,\bar{\sigma}_{12}\,\dot{\alpha} + [\bar{\sigma}, \bar{\tau}]_{12} + \tfrac{1}{2}[\dot{S}_{12}{}^{o} + 3\,\dot{\alpha}\,S_{12}{}^{o}] = 0 \tag{26}$$

Comparing Eq. (26) with (25a), it is immediately seen that the term involving torsion (fourth term in brackets) must be zero. Thus, the fourth term must satisfy the identity

$$[\dot{S}_{12}{}^{o} + 3\,\dot{\alpha}\,S_{12}{}^{o}] = 0 \tag{27}$$

which allows us to solve for the torsion as a function of time:

$$S_{12}{}^{o} = S_{12}{}^{o}(0)\,e^{-3\alpha}\ . \tag{28}$$

Examining field equation [13], an identity can immediately be made by recalling that

$$S_{12}{}^{o} = \tfrac{1}{2}\kappa\rho\; s_{12}\; u^{o} = \tfrac{1}{2}\kappa\rho\; s^{12}\; u^{o} = \tfrac{1}{2}\kappa\rho s^{12} \tag{29}$$

since $u^{\circ} = 1$. Thus, substituting Eq. (29) into the right-hand side of field equation [13] and subtracting from both sides, it can be written as

$$\overset{*}{\tilde\sigma}_{13} + 3\;\tilde\sigma_{12}\;\dot\alpha + [\tilde\sigma,\;\tilde\tau]_{13} - (3/2)\;S_{12}{}^{o}\;\tilde\tau_{23} = 0 \tag{30a}$$

while equation [31], using the same identity, can be written

$$\overset{*}{\tilde\sigma}_{31} + 3\;\tilde\sigma_{31}\;\dot\alpha + [\tilde\sigma,\;\tilde\tau]_{31} - \tfrac{1}{2}S_{12}{}^{o}\;\tilde\tau_{23} = 0 \tag{30b}$$

Since the shear and shear-vorticity commutator terms are symmetric in their indices, Eq. (30b) can be transformed into the following, by switching indices:

$$\overset{*}{\tilde\sigma}_{13} + 3\;\tilde\sigma_{13}\;\dot\alpha + [\tilde\sigma,\;\tilde\tau]_{13} - \tfrac{1}{2}S_{12}{}^{o}\;\tilde\tau_{23} = 0 \tag{30c}$$

To have consistency between Eqs. (30a) and (30c), set $\tilde\tau_{23} = 0$. Similar reasoning using field equations [23] and [32] yields $\tilde\tau_{13} = 0$.

Adding equations [11], [22], and [33] together and using the fact that Trace $[\tilde\sigma] = 0$ and $[\tilde\sigma,\;\tilde\tau]_{ii} = 0$, the resulting equation can be written as

$$-6\;\ddot\alpha - 9\;(\dot\alpha)^2 - (3/2)\;\tilde\sigma^2 = 3\kappa P + 2\kappa\rho\;\tilde\tau_{12}\;s^{12} \tag{31}$$

Field equation [00] is written as

$$3\;(\dot\alpha)^2 - \tfrac{1}{2}\tilde\sigma_{ij}\;\tilde\sigma^{ij} = \kappa\rho\;(1 + \varepsilon), \tag{32}$$

where, recalling from previous argument $[\rho' = \rho\;(1 + \varepsilon)]$, $\rho\;(1 + \varepsilon)$ can simply be replaced by ρ in the following equations. Thus, the time rate of change of the expansion can be solved as

$$(\dot\alpha)^2 = (1/6)\;\tilde\sigma^2 + (1/3)\kappa\rho. \tag{33}$$

Equation (33) can be substituted in Eq. (31) to yield

$$\ddot\alpha = -\kappa\tilde\sigma^2 - \tfrac{1}{2}\kappa(\rho + P) + (\kappa/3)\;\tau_{12}\;s^{12} \tag{34}$$

The expansion term α can be written in terms of an apparent rate of change of radius of the universe (Hubble expansion) as

$$\dot\alpha = \dot R/R \tag{35a}$$

$$\ddot\alpha = \ddot R/R - (\dot R/R)^2 \tag{35b}$$

Substituting the above plus writing the shear scalar as $\rho_\sigma = (1/2)\;\tilde\sigma^2$, Eq. (34) can be written as (let $\kappa = 1$);

$$\ddot{R}/R = -(2/3)\,\rho_\sigma - \rho/6 - P/2 + (1/3)\,\rho\,\tau_{12}\,S^{12} \tag{36}$$

To arrive at the appropriate power laws for each of the terms in Eq. (36), it is necessary to make use of several identities. Using the contracted Bianchi identities [9], it is possible to arrive at the first-order differential equation relating density and pressure (letting $P = \rho\gamma$). Then

$$\dot{\rho} = -3\,\dot{\alpha}\,(\rho + P) \tag{37a}$$

$$= -3\,\dot{\alpha}\,(1 + \gamma)\,\rho \tag{37b}$$

gives,

$$\rho = \rho_o\, e^{-3\alpha(1+\gamma)} \tag{38}$$

From Eq. (35a), $\exp(\alpha) = R$ can be written and then substituted in Eq. (38) to find

$$\rho = \rho_o\, R^{-3(1+\gamma)} \tag{39}$$

Because of the relationship between ρ and P, a power law can immediately be written for pressure P as

$$P = \tfrac{1}{2}\gamma\,\rho_o\, R^{-3(1+\gamma)} \tag{40}$$

The scaling for the shear scalar term, $\rho_\sigma = 1/2\tilde{\sigma}_{ij}\tilde{\sigma}^{ij}$, can be determined by the following analysis. From the form of the torsion chosen (spin vector aligned along the z-axis), the non-zero components of the coordinate shear is expressed in terms of a "shear ellipsoid" with the tetrads aligned such that [9]

$$\bar{\sigma}_{11} = \Sigma^o_{11}\,(1 + \cos\Phi) + \Sigma^o_{22}\,(1 - \cos\Phi) + 2\,\Sigma^o_{12}\,\sin\Phi \tag{41a}$$

$$\bar{\sigma}_{22} = \Sigma^o_{22}\,(1 + \cos\Phi) + \Sigma^o_{11}\,(1 - \cos\Phi) - 2\,\Sigma^o_{12}\,\sin\Phi \tag{41b}$$

$$\bar{\sigma}_{33} = \Sigma^o_{33} \tag{41c}$$

$$\bar{\sigma}_{12} = 2\,\Sigma^o_{12}\,\cos\Phi + (\Sigma^o_{11} - \Sigma^o_{22})\,\sin\Phi \tag{41d}$$

where

$$\Sigma^o_{ij} = (1/2)\,\bar{\sigma}^o_{ij}\,e^{-3\alpha} \tag{41e}$$

and

$$\Phi = 2\int \bar{\tau}_{12}\,dt \tag{41f}$$

The shear scalar can be represented as (sum of non-zero terms)

$$\bar{\sigma}^2 = \bar{\sigma}^2_{11} + \bar{\sigma}^2_{22} + \bar{\sigma}^2_{33} + 2\,\bar{\sigma}^2_{12} = 2(\bar{\sigma}^2_{11} + \bar{\sigma}^2_{22} + \bar{\sigma}^2_{12} + \bar{\sigma}_{11}\,\bar{\sigma}_{22}) \tag{42}$$

Performing the indicated operations obtains

$$\rho_\sigma = 4[(\sigma^o_{11})^2 + (\sigma^o_{22})^2 + (\sigma^o_{12})^2 + \sigma^o_{11}\,\sigma^o_{22}$$

$$+ \sigma^o_{12}\,(\sigma^o_{11} - \sigma^o_{22})\,\sin 2\phi] \tag{43}$$

In Eq. (43), replace all σ^o_{ij} by their corresponding $\tilde{\sigma}_{ij}$ terms,

$$\rho_\sigma = [(\tilde{\sigma}^o_{11})^2 + (\tilde{\sigma}^o_{22})^2 + (\tilde{\sigma}^o_{12})^2 + \tilde{\sigma}^o_{11}\,\tilde{\sigma}^o_{22}$$

$$+ \tilde{\sigma}^o_{12}\,(\tilde{\sigma}^o_{11} - \tilde{\sigma}^o_{22})\,\sin 2\phi]\,e^{-6\alpha} \quad . \tag{44}$$

Thus, the term ρ_σ scales as R^{-6}.

To determine the scaling laws for the shear term τ_{12} involved in Eq. (36), use the result of Reference 9 which shows by dimensional analysis that the shear evolution is related to the proper time T by the relationship

$$\tilde{\tau}_{ij} = \tau^o_{ij}\,T^{-1} \quad . \tag{45}$$

To get a scaling relationship between R and time T, proceed as follows:

The field equation [00] can be written as

$$(\dot{R}/R)^2 = (1/3)\,\rho^o_\sigma\,R^{-6} + \rho_o\,R^{-3(1+\gamma)} \quad . \tag{46}$$

For $\gamma = 0$ (dust), the last term is R^{-3}, for $\gamma = 1/3$ (radiation) it is R^{-4}, and for $\gamma = 1$ (stiff matter) it is R^{-6}. Depending on the region of interest (R > or < 0) and the value of γ, one or the other term in Eq. (46) will dominate. As an example, for dust ($\gamma = 0$) and R > 1, the second term will dominate. Thus, the differential equation in Eq. (46) is approximated and written as

$$(\dot{R}/R)^2 = \rho_o\,R^{-3} \quad . \tag{47}$$

Solving for R, the following is obtained:

$$R^{3/2} = (3/2)\,(\rho_o)^{\frac{1}{2}}(T - T_o) \quad , \tag{48a}$$

which indicates that R scales with respect to time as

$$R \sim T^{2/3} \tag{48b}$$

Proceeding similarly for the other values of γ, both for R > 1 and R < 1, similar scaling relationships are derived between R and T which are summarized in Table 4.

TABLE 4. Scaling Relationships

Type Matter	Dominant Term	R Values	Scaling Law
Dust: $\gamma = 0$	$1/3\ \rho_\sigma^o R^{-6}$	$R < 1$	$R \sim T^{1/3}$
	$\rho^o R^{-3}$	$R > 1$	$R \sim T^{2/3}$
Radiation: $\gamma = 1/3$	$1/3\ \rho_\sigma^o R^{-6}$	$R < 1$	$R \sim T^{1/3}$
	$\rho^o R^{-4}$	$R > 1$	$R \sim T^{1/2}$
Stiff Matter: $\gamma = 1$	$1/3\ (\rho_\sigma^o + \rho_o)R^{-6}$	$R < 1$	$R \sim T^{1/3}$
	$1/3\ (\rho_\sigma^o + \rho_o)R^{-6}$	$R > 1$	$R \sim T^{1/3}$

As seen, for all matter types and $R < 1$, $R \sim T^{1/3}$. Thus, T goes as R^3 and the scaling law for shear is written as

$$\bar{\tau}_{12} = \tau_{12}^o R^{-3} \quad . \tag{49}$$

To complete the scaling laws for all the terms involved in Eq. (36), the scaling properties for the spin density s^{12} need to be postulated. In order to do this, the first-order differential equation relating the time evolution of spin density to tetrad rotation rate and as spin density is written [7]

$$D(s_{ij}) = w_{\ell i}\, s^{\ell}{}_{j} + w_{j\ell}\, s^{\ell}{}_{i} \tag{50}$$

which shows that the time derivative of spin density is proportional to terms like $w_{i\ell}\, s^{\ell}{}_{j}$. Thus, a differential equation relating the time rate of spin and spin and tetrad rotation may be written as

$$\dot{s} = w\, s \ , \tag{51a}$$

which gives ($w = -\tau = \tau^{o}/T$)

$$ds/s = -\tau^{o}\, dT/T \quad . \tag{51b}$$

Solving for s, gives

$$s_{12} \sim 1/T \sim R^{-3} \tag{51c}$$

Combining the scaling laws for density, shear, and spin, the following is obtained:

$$(1/3)\ \rho\ \tau_{12}\ s^{12} = (1/3)\ \rho_o\ \tau^{o}_{12}\ s^{o}_{12} R^{-3(3+\gamma)} \tag{51d}$$

Combining Eqs. (39), (40), (44), and (51d) in Eq. (36), derives

$$\ddot{R}/R = -(\gamma/2 + 1/6)\ \rho_o\ R^{-3(1+\gamma)} - (2/3)\rho^{o}_{\sigma}\ R^{-6}$$

$$+ (1/3)\rho_o\ \tau^{o}_{12}\ s^{o}_{12}\ R^{-3(3+\gamma)} \quad . \tag{52}$$

The usual relationships between matter density and pressure used in cosmological models are non-interacting dust ($\gamma = 0$), radiation ($\gamma = 1/3$), and so-called "stiff" matter ($\gamma = 1$). For dust, radiation, and stiff matter, the term containing shear and spin density scales as R^{-9}, R^{-10}, and R^{-12}, respectively, is immediately concluded. By putting in the respective values for γ into Eq. (52a), the following is obtained for dust ($\gamma = 0$):

$$\ddot{R} = -(1/3)\ \rho_o\ R^{-2} - (2/3)\ \rho^{o}_{\sigma}\ R^{-5} + (1/3)\ \rho_o\ \tau^{o}_{12}\ s^{o}_{12}\ R^{-9} \tag{53a}$$

for radiation ($\gamma = 1/3$):

$$\ddot{R} = -(1/3)\ \rho_o\ R^{-3} - (2/3)\ \rho^{o}_{\sigma}\ R^{-5} + (1/3)\ \rho_o\ \tau^{o}_{12}\ s^{o}_{12}\ R^{-10} \quad , \tag{53b}$$

and, for stiff matter ($\gamma = 1$):

$$\ddot{R} = -(2/3)\ (\rho_o - \rho^{o}_{\sigma})R^{-5} + (1/3)\ \rho_o\ \tau^{o}_{12}\ s^{o}_{12}\ R^{-12} \quad . \tag{53c}$$

As seen from Eqs. (53a, b, and c), for small values of R (less than one in normalized coordinates), the spin density term will dominate as a large positive term, thus providing the source for increasing expansion; shear does not effectively dampen the expansion. According to this model, a radiation dominated, early epoch cosmology will have its expansion strongly driven by the spin kinetic energy of the fluid.

IV. CONCLUSIONS

By exact solution, it is shown that expansion, in the early radiation dominated phases of a Bianchi Type I Einstein-Cartan cosmology, is driven positively by the spin kinetic energy of a perfect fluid using the improved SEMT of Ray and Smalley. It is immediately obvious that $R = 0$ is not a solution to either Eq. (53a, b, or c). A solution set to these equations can be found, but their exact form depends on the boundary conditions. Even in this simple model, it is concluded that the spin-energy not only drives the expansion, but prevents the occurrence of the singularity and causes the apparent radius of the universe to "bounce".

Gasperini [5] has demonstrated a spin-driven inflation using a time-averaging and scaling analysis of the Einstein-Cartan equations. Kopczynski [10] and Trautman [11] have obtained minimal radius solutions for torsion cosmological models containing polarized dust.

When going to an Einstein-Cartan cosmology with spin density using the Ray-Smalley improved SEMT of spinning fluids, it is apparent from this simple model that the properties of the standard Bianchi Type I cosmology are drastically changed. Thus, it could be argued that astronomical observations which lead one to classify behavior as a Bianchi Type of higher number in standard general relativistic theory, could, in reality, be torsion "masquerading" itself as some sort of pseudo-curvature in a universe which obeys the Einstein-Cartan formalism. Much work remains to be done in this area, including a systematic re-examination of cosmological properties of all Bianchi Type structures within the framework of the correct description of spinning fluids in the torsion theory of gravitation.

REFERENCES

1. Linde, A. D., Rep. Prog. Phys., 47, 925 (1984); Brandenberge, R., Rev. Mod. Phys., 57, 1 (1985).

2. Abbott, L. F., and Wise, M. B., Nucl. Phys., B224, 541 (1984); Astrophys. J., 282, 142 (1984); Phys. Lett., 135B, 279 (1984).

3. Gron, O., Phys. Rev., D32, 2522 (1985); 1586 (1985); D33, 1204 (1986); J. Math. Phys., 27, 1490 (1986); Ellis, J., and Olive, K. A., Nature, 303, 679 (1983).

4. Fabbri, R., Lett. Nuovo Cim., 26, 77 (1979); Wald, R. M., Phys. Rev., D28, 2118 (1983); Webe, E., J. Math. Phys., 25, 3279 (1984).

5. Gasperini, M., Phys. Rev. Lett., 56, 2873 (1986).

6. Misner, C. W., Astrophys. J., 151, 431 (1968).

7. Ray, J. R., and Smalley, L. L., Phys. Rev. Lett., 49, 1059 (1982).

8. Ryan, M. P. and Shepley, L. C., Homogeneous Relativistic Cosmologies, (Princeton University Press, 1975).

9. Fennelly, A. J., J. Math. Phys., 22, 126 (1981).

10. Kopczynski, W. Phys. Lett., 39, 219 (1972).

11. Trautman, A., Nature (London), Vol. 242, 7 (1973).

DISTRIBUTION

	No. of Copies
US Army Materiel System Analysis Activity ATTN: AMXSY-MP Aberdeen Proving Ground, MD 21005	1
ITT Research Institute ATTN: GACIAC 10 W. 35th Street Chicago, IL 60616	1
AMSMI-RD, Dr. McCorkle	1
Dr. Rhoades	1
-RD-GC, Dr. Yates	1
-RD-GC-T, Mr. Alongi	1
-RD-GC-T, Mr. Plunkett	1
Dr. Bradas	5
-RD-CS-R	15
-RD-CS-T	1
-GC-IP, Mr. Bush	1

END

10-87

DTIC